Combined Three-Dimensional Finite-Difference and Integral Method

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Theme

A combined integral, explicit finite-difference method is presented. The example calculation predicts the three-dimensional compressible blast wave loading on a rigid four-sided structure. Previous techniques for the inviscid slip flow pressure loading on such structures include a variety of linear interpolation and extrapolation techniques to predict the transient pressure distribution on the structure walls. The present approach introduces a combination of the finite-difference computation with an integral method[3,4] which is confined to the irregular finite-difference cell volumes adjoining the walls, edges, and corners of the structure. Separate numerical techniques are applied inturn to the finite-difference computations and the integral regions, with an initial condition boundary condition overlap coupling at each discrete time step in the computations.

Contents

A rigid structure, shown cross-hatched (Fig. 1) is immersed in transient blast wave flow emanating from a distant ground-level, $K=1$, source. The finite-difference computation is implemented on an orthogonal three-dimensional mesh, with regular uniform mesh spacing equivalent to ¼ the characteristic height, h, of the structure as shown. The use of variable mesh spacing and taking advantage of the centerline symmetry along the flow direction were considered but not applied for reasons discussed in the back-up paper.

In inviscid, compressible, adiabatic flow, the governing Eulerian equations are written vectorially:

Continuity

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho U) \tag{1}$$

Momentum

$$\frac{\partial \rho U}{\partial t} = -\nabla \cdot (p\bar{\bar{\delta}} + \bar{\bar{q}}) - (\rho U \nabla) U \tag{2}$$

Energy

$$\frac{\partial \rho E}{\partial t} = -\nabla \cdot [(p\bar{\bar{\delta}} + \bar{\bar{q}}) \cdot U + \rho E U] \tag{3}$$

State

$$p = (\gamma - 1)\rho e, \quad \text{where } e \equiv E - (U \cdot U)/2 \tag{4}$$

The variables ρ, U, P, $\bar{\bar{q}}$, and E are the mass density, vector velocity, pressure, von Neumann-Richtmyer numerical damping, and specific total energy, respectively. The polytropic coefficient, γ, is held constant in the present calculation, while $\bar{\bar{\delta}}$ denotes the unit tensor.

The basic finite-difference procedure makes use of a first-order space and time accurate scheme for three-dimensional compressible flow.[1] In this method conditional Courant stability is achieved by use of a spatial staggering of the velocity computational nodes from the mass (and energy) computational nodes.[2] The boundary conditions applied include assumptions of a thermally insulated (slip-flow) ground surface, insulated slip-flow at the structure walls, and continuous flow-through conditions at the outer boundaries (I_{max}, J_{min}, J_{max}, K_{max}). The unsteady incident shock-wave is conditionally imposed at the input boundary (I = 1).

To predict the transient pressure loading on the walls of the structure, an integral method is applied to the finite-difference cells which overlap or adjoin the structure. Dimensionless variables are introduced relative to the integral domain coordinate system (see Fig. 2).

$$\xi, \eta, \zeta = (1/\zeta^0)(\xi_0, \eta^0, \zeta^0)$$

$$f, g, h = (1/U^0_\infty)(\dot{\xi}_0, \dot{\eta}^0, \dot{\zeta}^0)$$

$$r = \rho^0/\rho^0_\infty \; p = p^0/(\rho^0_\infty U^{02}_\infty) E = E^0/(U^{02}_\infty) \tag{5}$$

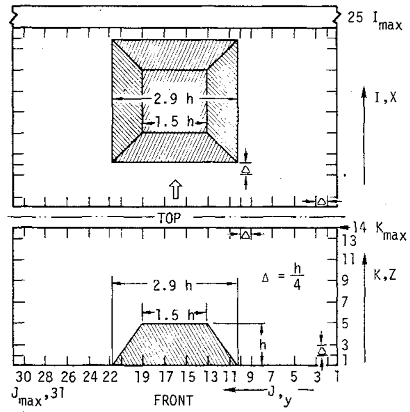

Fig. 1 Three-dimensional computational grid used in finite-difference simulation.

Presented at the AIAA 2nd Computational Fluid Dynamics Conference, Hartford, Connecticut, June 14-20, 1975. (No preprints, pp. 29-37 bound volume Conference papers); Received June 20, 1975; synoptic received November 12, 1975; revision received December 22, 1975. Full paper available from AIAA Library, 750 Third Avenue, New York, N.Y. 10017. Price: Microfiche, $2.00; hard copy, $5.00. **Order must be accompanied by remittance.** Supported in part by Defense Nuclear Agency under Contract DNA-001-73-C-0167, and, in part, by the U.S. Energy Research and Development Administration under Contract W-7405-ENG-48.

Index categories: Shock Waves and Detonations; Supersonic and Hypersonic Flow; Computer Technology and Computer Simulation Techniques.

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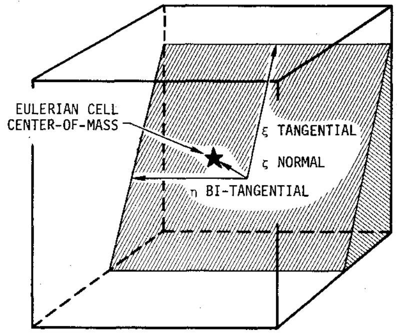

Fig. 2 Integral domain and coordinates.

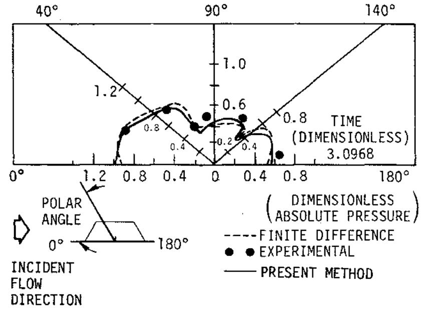

Fig. 3 Meridional plane distribution at late time.

The subscript "∞" on the velocity moduli designates maximum initial blast wave velocity amplitudes, while the same subscript on the density and energy terms denotes quiescent conditions at the structure before blast wave arrival. The subscript "δ" refers to the "outer" finite-difference results applied as boundary conditions at every discrete time step. Integration along the outward normal of the governing equations in the transformed domain yields the integro-differential relations for the slip wall boundary calculations:

Mass

$$\frac{\partial r}{\partial t}+\frac{\partial I_6}{\partial \xi}+\frac{\partial I_7}{\partial \eta}+(rh)_\delta=0 \tag{6}$$

ξ Momentum

$$\frac{\partial rf}{\partial t}+\frac{\partial I_0}{\partial \xi}+\frac{\partial I_1}{\partial \eta}+(rfh)_\delta+\left[\frac{\partial p}{\partial \xi}\right]_\delta=0 \tag{7}$$

η Momentum

$$\frac{\partial rg}{\partial t}+\frac{\partial I_2}{\partial \xi}+\frac{\partial I_3}{\partial \eta}+(rgh)_\delta+\left[\frac{\partial p}{\partial \eta}\right]_\delta=0 \tag{8}$$

$$\frac{\partial rE}{\partial t}+\frac{\partial I_4}{\partial \xi}+\frac{\partial I_5}{\partial \eta}+\{h(E+p)\}_\delta=0 \tag{9}$$

where $p=p+q_{ii}$, where q_{ii} is the diagonal component of the damping tensor $\bar{\bar{q}}$.

The integrals I_i are defined by

$$I_0\equiv\int_0^1(rf^2)\,d\zeta,\ I_1\equiv\int_0^1(rfg)\,d\zeta,\ I_2\equiv I_1$$

$$I_3\equiv\int_0^1(rg^2)\,d\zeta,\ I_4\equiv\int_0^1\{g(E+p)\}\,d\zeta,\ I_5\equiv\int_0^1\{g(E+P)\}\,d\zeta$$

$$I_6\equiv\int_0^1(rf)\,d\zeta,\ I_7\equiv\int_0^1(rg)\,d\zeta \tag{10}$$

Boundary conditions over the integral domain are the instantaneous conditions for the outer (1) limit (supplied by the finite-difference calculation), while the wall is the inner (0) limit.

The wall solutions of Eqs. (6-9), subject to the boundary conditions, are obtained at each finite-difference time step by substitution of quadratic polynomial functions for the dimensionless variable distributions in the integrals, Eq. (10). Solution is by means of an implicit finite-difference scheme at the wall surface where the polynomial coefficients are determined as part of the iterative matrix inversion procedure.[1] The wall solution scheme is second-order accurate in space and first-order accurate in time.

The present method provides smoothness in the results that is not present when using ordinary linear extrapolation procedures. This is primarily a result of the simultaneous coupling of tangential and normal gradients at the wall through the integral approximation.

In the back-up paper the results of the combined integral-explicit finite-difference method are compared with both basic finite-difference and shock tube experiments.[5] Typical results of the enhanced accuracy of the present procedure are shown in Fig. 3, where the instantaneous experimental values are depicted by the solid symbol.

References

[1]Buckingham, A.C. and Birnbaum, N.K., "Transient Computation of Rigid Structure Air Loads in Three Dimensions," *Computational Methods in Nonlinear Mechanics,* Texas Institute for Computational Mechanics, Austin, 1974, pp. 509-518.

[2]Noh, W.F., "CEL: A Time-Dependent Two-Space Dimensional Coupled Eulerian-Language Code," *Methods in Computational Physics,* Vol. 3, Academic Press, New York, 1974, pp. 117-179.

[3]Maslen, S.H. and Moeckel, W.E., "Inviscid Hypersonic Flow Past Blunt Bodies," *Journal of Aerospace Sciences,* Vol. 24, No. 9, Sept. 1957, pp. 683-693.

[4]Hoshizaki, H., "Convective and Radiative Heat Transfer to Blunt Bodies at Angle of Attack," Lockheed Palo Alto Research Lab. Rept. 4-06-66-11, June 1966, Palo Alto, Calif.

[5]Dunbar, G.R., Moore, E.T., Mumma, D., Buckingham, A.C., "Simulation of Air Blast over an SDU Turret Structure: Vol. 2 Shock Tube Experiments (U)," Physics International Co. Rept. DNA 3461 F-2, Sept. 1974, San Leandro, Calif.